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UNITED STATES DEPARTMENT OF THE INTERIOR BUREAU OF MINES HELIUM ACTIVITY HELIUM RESEARCH CENTER INTERNAL REPORT

STEADY-STATE LAMINAR FLOW BOUNDARY CONDITIONS

FOR A STAINLESS STEEL COILED CAPILLARY VISCOSIMETER

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R. A. Guereca, H. P. Richardson, and L. M. Walker

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STEADY-STATE LAMINAR FLOW BOUNDARY CONDITIONS FOR A STAINLESS STEEL COILED CAPILLARY VISCOSIMETER

by

R. A. Guereca, $\frac{1}{}$ H. P. Richardson, $\frac{2}{}$ and L. M. Walker $\frac{3}{}$

ABSTRACT

An analysis to determine criteria for steady-state, laminar flow boundary conditions for a coiled-tube gas viscosimeter is made by comparing experimental data from a thick-walled, stainless steel capillary in a horizontal, straight-tube condition with similar data taken after the same capillary was formed into a helix. Overlapping volumetric flowrates and pressure drops are covered for both configurations at pressure levels of 28, 225, and 1,000 psia at 300° K. Friction factor, Reynolds number, and Dean number correlation plots show conditions where the coiled-tube flow deviates from straight-tube flow. The coiled-tube data indicate departure from Poiseuille steady-

^{1/} Supervisory research chemist, project leader, physical properties studies, Helium Research Center, Bureau of Mines, Amarillo, Tex.

^{2/} Research chemist, Helium Research Center, Bureau of Mines, Amarillo, Tex.

^{2/} Physical Sciences Library, University of Kansas, Lawrence, Kan.; formerly research chemist, Helium Research Center, Bureau of Mines, Amarillo, Tex.

Work on manuscript completed October 1966.

STEADY-STATE LAMINAR FLOW BOUNDARY CONDITIONS FOR A STATISTICS STEEL COLLED CAPILLARY VISCOSIPETER

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R. A. Guerece, L. H. P. Hichardson, E. and L. M. Welker L.

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^{3/} Physical Sciences Library, University of Kansus, Lawrence, Man .: formerly research chemist, Wellum Research Center, Burcan of Mines, Amarillo, Tax.

state laminar flow at a Reynolds number close to 390 and a Dean number of about 15. Below these conditions and within the accuracy of the data reported, Dean's circulation is negligible, for all practical purposes. It is suggested that for coiled systems in general, an upper limit for steady-state laminar flow appears to be at a Dean number of about 10.

INTRODUCTION

In an earlier publication (7), $\frac{4}{}$ the Bureau of Mines Helium

Research Center presented accurate measurements of physical dimensions of a 19-foot section of stainless steel capillary tubing and developed a working equation for a gas viscosimeter to include the effects of temperature and pressure on the capillary dimensions. In a later publication (10), procedures were derived to correct for non-uniformity of various capillary-bore shapes; it was shown that the bore of the 19-foot capillary section was practically uniform.

This report presents criteria for establishing conservatively chosen Reynolds (Re) and Dean (Dn) numbers below which Poiseuille steady-state laminar flow apparently exists without perturbations due to secondary circulation known as Dean's circulation (4, 5, 12).

^{4/} Underlined numbers in parentheses refer to items in the list of references at the end of this report.

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Dean (4, 5) was the first to study the coiling problem from a theoretical viewpoint. His results indicated that the ratio of the mean velocities in a straight and a curved tube of the same dimensions is a function of

Re
$$(d/D)^{1/2}$$
 (1)

which is the Dean number; d is the capillary-bore internal diameter and D is twice the average radius of curvature of the helix. White (13) was the first to experimentally study Dean's relationship by plotting the friction factor ratio of the coiled-to-straight configuration against the Dean number for D/d ratios of 2,050, 50, and 15, the latter being the most tightly coiled system. In White's helices, the appearance of fully developed turbulence occurred later than in the corresponding straight tube and was at increasing Reynolds numbers as the D/d ratio decreased; however, the coil data deviated from strictly Poiseuille straight-tube laminar flow (64/Re) at decreasing Reynolds numbers. recently, Creutz (3) determined the effect of coiling and apparent lack of fully developed turbulence for a wide range of D/d ratios. results indicate that fully developed turbulence occurs at progressively smaller values of D/d. For porous media, the limiting practical case, the generally accepted Re for staady-state laminar flow is unity (11). Therefore, one might expect that coil fluid flow would start deviating from strictly straight-tube laminar flow at a Reynolds number between 1 and about 2,200, the Re representing the beginning of turbulence in a straight-tube system. It also would be expected that the departure from

Re (8/10) 1/2

(3)

strictly straight-tube flow would occur sooner with helices that were more tightly wound (smaller D/d ratios). Theoretically, a curved tube should always be more resistant to fluid flow that the corresponding straight tube; the slight difference, however, would have required much finer measurements than those employed here and very probably would have been absorbed in the friction factor.

Berwald and Johnson (1) reported gas viscosities of 25 natural gases, using a coiled viscosimeter; according to Carr (2), their data are questionable because of a maximum variation of 25 percent in the results reported. Accuracies of viscosity values, per se, are not discussed in this report (our apparent viscosities are internally consistent to at least two percent). However, rapid changes in apparent viscosity are mentioned in discussing the experimental results where the coiled-tube flow deviates from strictly straight-tube flow. The secondary parameters of entrance, kinetic energy, gas slippage, and gas compressibility effects, although important for absolute gas viscosity determinations, were considered negligible in dealing with friction factors, Reynolds numbers, and Dean numbers for a capillary of the dimensions used and at the flowrates and pressure drops of these experiments. In no case were the experiments run in the diffuse flow region.

EXPERIMENTAL APPARATUS AND PROCEDURES

The apparatus, figure 1, consisted of a closed, constant-volume

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Server, value and Johnson (1) required gas viscosities of 2) natural gates of each value, value a relief viscosite value of the account of the percent of the results reported, accounted of viscosity values, per se, are not discussed in this reported, accounted of viscosity values, per se, are not discussed in this report (our apparent viscosities, are internally consistent to at least two persons). However, repre charges to apparent viscosity are meathoned in discussing the experimental results where the costs of the flow deviates from striking the experimental results where the consider the flow deviates of entrance, kineral energy, see alignate, and was compressed to the final day and the same that was compressed to the final day and the same time and the same visionity.

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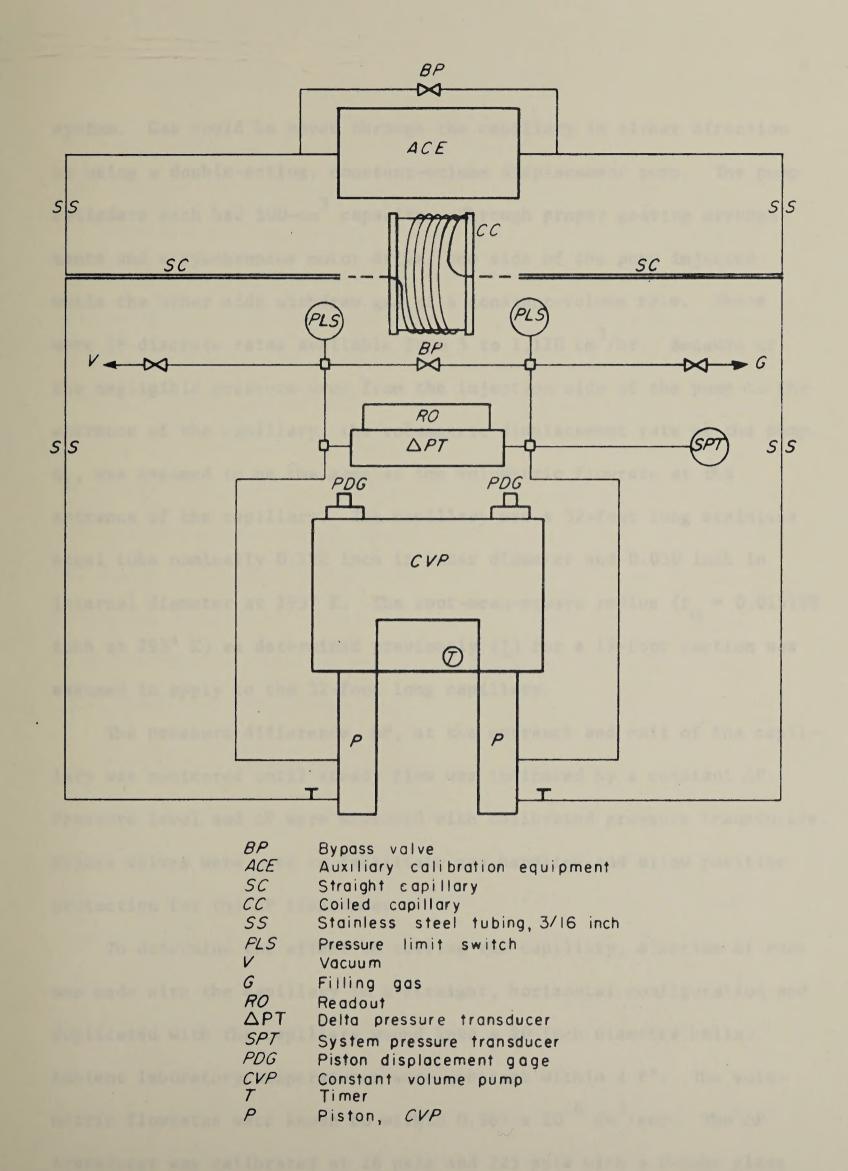


FIGURE 1.- Schematic Diagram of Experimental Apparatus.

system. Gas could be moved through the capillary in either direction by using a double-acting, constant-volume displacement pump. The pump cylinders each had 500-cm^3 capacity. Through proper gearing arrangements and a synchronous motor drive, one side of the pump injected while the other side withdrew gas at a constant-volume rate. There were 28 discrete rates available from 5 to 1,120 cm 3 /hr. Because of the negligible pressure drop from the injection side of the pump to the entrance of the capillary, the volumetric displacement rate of the pump, Q_1 , was assumed to be the same as the volumetric flowrate at the entrance of the capillary. The capillary was a 32-foot long stainless steel tube nominally 0.122 inch in outer diameter and 0.030 inch in internal diameter at 293° K. The root-mean-square radius ($r_m = 0.015199$ inch at 293° K) as determined previously ($r_m = 0.015199$ inch at 293° K) as determined previously ($r_m = 0.015199$ to the 32-foot long capillary.

The pressure difference, $\triangle P$, at the entrance and exit of the capillary was monitored until steady flow was indicated by a constant $\triangle P$. Pressure level and $\triangle P$ were measured with calibrated pressure transducers. Bypass valves were used to facilitate gas handling and allow positive protection for the $\triangle P$ transducer.

To determine the effect of coiling the capillary, a series of runs was made with the capillary in a straight, horizontal configuration and duplicated with the capillary wound into a 20-inch diameter helix. Ambient laboratory temperatures were constant within 2 F°. The volumetric flowrates were known to within 0.965 x 10^{-6} cm $^3/\text{sec}$. The ΔP transducer was calibrated at 28 psia and 225 psia with a U-tube glass

system. One could be soved through the capitlary in either direction by using a double-scring, constant-wolume displacement pump. The pump of these each bed 500-cm upacity. Through proper genting arrangements and a synchronous water drive, one side of the pump injected while the other side withdraw gas at a constant-value rate. There were 35 discrete rates available from 1 to 1,120 cm thr. Became of the regisfible trasseure drop from the injection side of the poop to the entrance of the capitlary, the volumetric displacement was of the ramp of the ramp anticapte of the capitlary, the volumetric displacement at the authorized to be negligible. The repulse was a 32-feet long staining areal tabe negligible to 127 inch in outer of sear-square ratio (n = 8.0539) attends at 293° E) as determined previously (2) for a 19-feet section was shown at the time at 293° E) as determined previously (2) for a 19-feet section was small one apply to the 32-feet long capitlary.

The pressure difference, AP, at the purpose and exit of the capilfory was mentioned until steady flow was indicated by a constant LP.

Pressure level and AP were measured with calibrated pressure transducers

Bypass valves were used to facilitate gas handling and allow positive

protection for the AP transducer.

To determine the effect of colling the capillary, a saries of come wes made with the capillary in a-straight, horizontal configuration and duplicated with the capillary wound into a 20-inch diameter helix.

Ambient laboratory temperatures were constant within 2 P*. The volumetric flowrotes were known to within 0.955 x 10⁻⁶ co /sec. The AP transducer was colibrated at 25 pais and 225 pais with a U-the glass

manometer containing n-butyl alcohol; transducer accuracy was at least 0.01 psi for all pressure drops and reproducible to 0.001 psi.

Nitrogen gas was used at pressure levels of 28, 225, and 1,000 psia, all at 300° K. For the 1,000-psia run, the 225-psia calibration was used, although there was evidence of a small zero-shift between pressure calibration levels. At given flowrates, Q_1 , $\triangle P$ was reproducible to 0.001 psi; this was confirmed by plotting $\triangle P/L_T$ versus Q_m , where L_T was the corrected length at 300° K, and Q_m was the computed average volumetric flowrate (7). The mean radius, r_m , was corrected for temperature and pressure level changes, also shown in (7). Due to mechanical difficulties after the straight-tube run at 225 psia, the capillary tubing was reworked so that at the 1,000-psia run, the length was slightly less; in reworking the tubing for the coiled runs, another short length was cut off the capillary.

SUMMARY AND DISCUSSION OF RESULTS

Log-log plots of friction factor versus Reynolds number for both the straight and coiled configurations, and log-log plots of the friction factor ratio (coiled to straight) versus the Dean number are used to correlate experimental results. Tables 1 and 2, and figures 1a and 2

Figure la.-Steady-state boundary flow conditions, 1,000 psia and 300° K, friction factor versus Reynolds number.

Figure 2. -Steady-state boundary flow conditions, 1,000 psia and 300° K, friction factor ratio versus Dean number.

dentities of the set pressure drops and reproductate to 0 001 per

Nitrogen par was used at pressure levels of 10, 225, and 1,000 pairs all at 100" E. For the 1,000-pair run, and 225-pair calibration was cared, although chars was evaluated of a small zero-shift between pressure calibration levels. At given fluorated, Q, 25 was reproductive to 0.001 cat, this was confirmed by plotting CP+L, versus Q, where Ly was the compared descape was the compared descape was the compared descape value at the tention of 10 and the compared descape restored was the compared for tentioning was pressure to the standard of the persons and pressure to the standard to 1). Our to mean the tubing was resorbed so that at the 1,000-main ton at 250 pairs, the capitles alignments for the tention at the length was cut off the copillary that the tention call the copillary and another short tention that the copillary and another short tention to the tention was alightly have to fit the capillary

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Figure la Steady-state houselfs flow conditions, 1,000 pala and 300° K.

Figure 2. -Steady-state bookdary first conductions, 1.000 from and 100° E.

TABLE 1.-Experimental results, 1,000 psia and 300° K, coiled capillary

Q _m , cm ³ /sec	ΔP/Q _m	Apparent viscosity, N, micropoises	Reynolds number, Re	Dean number, <u>Dn</u>	f _s , 64/Re	f_c	f _c /f _s
0.016666	3.072	191.7	113.028	4.42	0.56624	0.57203	1.0102
.019445	3.070	191.6	131.869	5.16	.48533	.48998	1.0096
.022223	3.051	190.4	150.709	5.90	.42466	.42608	1.0033
.027778	3.046	190.1	188.385	7.37	.33973	.34030	1.0017
.033335	3.060	191.0	226.069	8.85	.28310	. 28487	1.0063
. 044447	3.051	190.4	301.429	11.80	.21233	.21302	1.0032
.055560	3.060	191.0	376.793	14.75	.16986	.17091	1.0062
.066673	3.129	195.3	452.160	17.70	.14155	.14565	1.0290
.077787	3.173	198.0	527.530	20.65	.12133	.12659	1.0434
.088901	3.233	201.8	602.905	23.60	.10615	.11284	1.0630
.111131	3.364	209.9	753.670	29.51	. 08491	.09393	1.1062
.133364	3.491	217.8	904.443	35.41	.07076	.08124	1.1481
.177836	3.718	232.0	1,206.040	47.22	. 05306	.06488	1.2228
.222319	3.927	245.1	1,507.717	59.03	. 04245	.05482	1.2914

Mean radius, $r_m = 0.0152017$ inch = 0.0386123 cm

Total length, $L_T = 964.210$ cm

Viscosity,
$$\eta = \frac{\pi r_{\text{m}}^4}{8L_{\text{T}}} \cdot \frac{\Delta P}{Q_{\text{m}}} = 62.41750 \frac{\Delta P}{Q_{\text{m}}}$$
, micropoises; $Q_{\text{m}} = Q_1(1 + \frac{\Delta P}{2P_{\text{m}}})$

Density, $\rho = 0.07807 \frac{g}{cm^3}$, interpolated from Flynn (6)

Reynolds number, Re =
$$\frac{2\rho r_m}{\eta^* A} Q_m = 6781.767 Q_m$$
; A = πr_m^2 , cm²; $\eta^* \cong 190$ micropoises, from Flynn (6)

Dean number, Dn = Re
$$\sqrt{\frac{2r_m}{D}}$$
 = 0.039149 Re; D = helix diameter = 50.3865 cm

Friction factor, $f_s = \frac{64}{Re}$ = theoretical friction factor, straight tube

Friction factor,
$$f_c = \frac{4r_m A^2}{\rho L_T} \cdot \frac{\Delta P}{Q_m^2} = 3.103468 \times 10^{-3} \frac{\Delta P}{Q_m^2} = \text{experimental friction}$$

TABLE 1. -Experi-wated passits, 1,000 pate and 300° K. colled capillary

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1,010.1					9.072	
a080.1	1.000					
	30, 400					
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1,011						
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	08124				184.8	ABREEU.
						028557
Alec-1	58420.				458.€	

Mean redies. r . 0 01:2017 tech = 0 0385123 cm

Total langth, L. = 264.210 cm

Density, p = 0.07807 Ep (miorpolatud from Fiven (6)

Bean marker Dn = Re: \ - 0.039149 Re; D = 5milk diemeter = 90.3865 am

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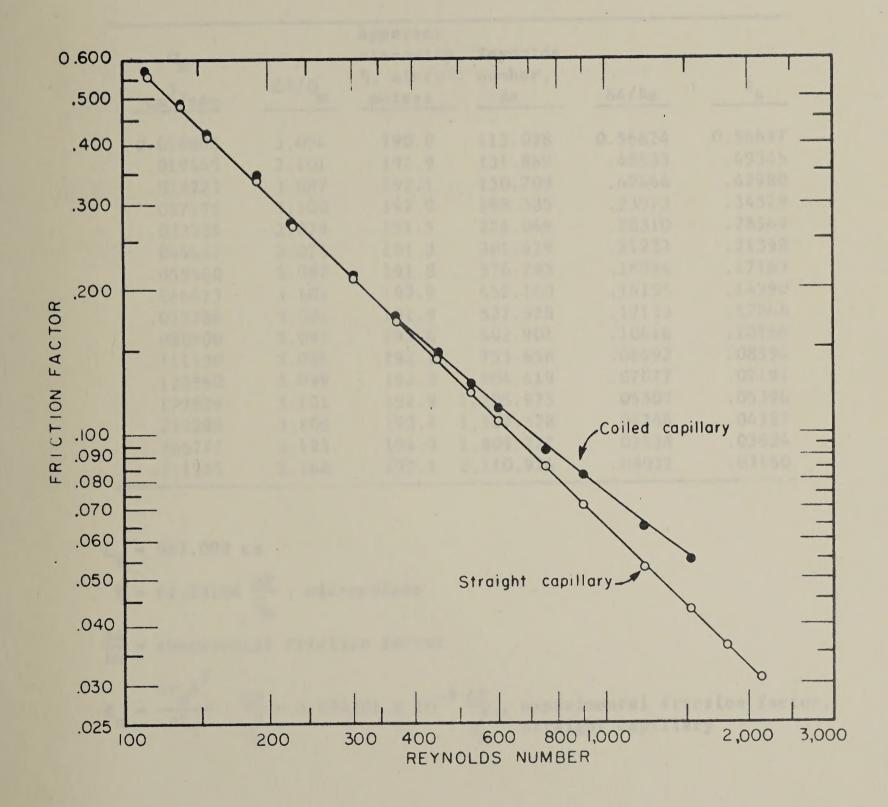


FIGURE 1a.—Steady—State Boundary Flow Conditions, 1,000 psia and 300 °K, Friction Factor Versus Reynolds Number.

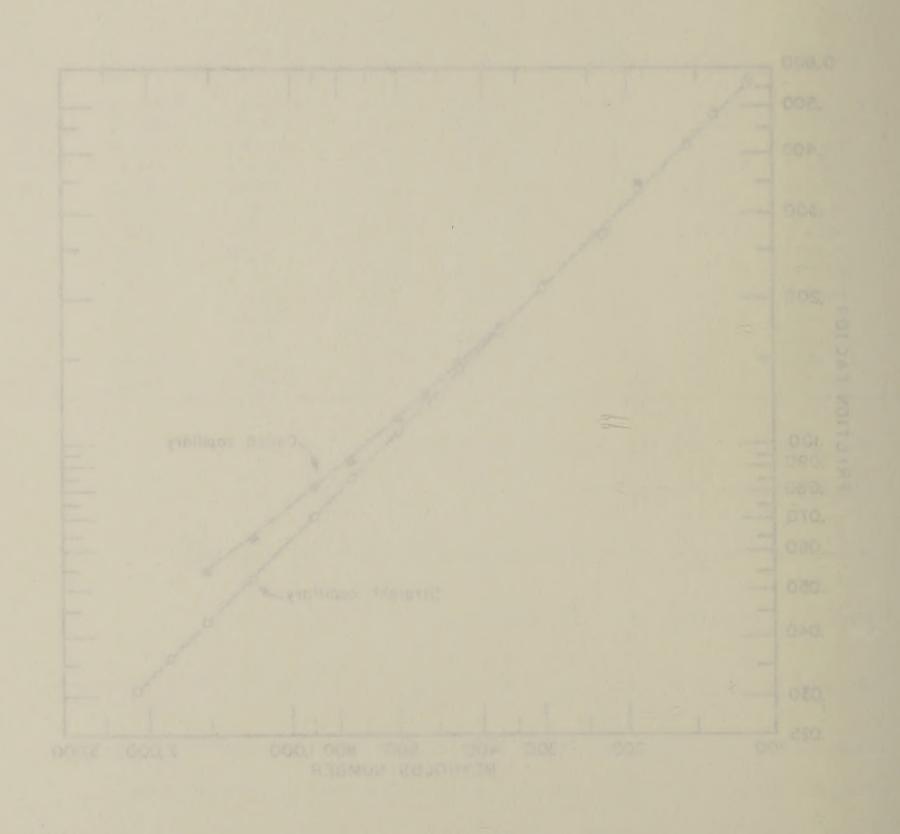


FIGURE In-Steady-State Soundary Flow Conditions, (200 paid

TABLE 2.-Experimental results, 1,000 psia and 300° K, straight capillary

Q _m , cm ³ /sec	ΔP/Q _m	Apparent viscosity N, micropoises		64/Re	fs
0.016666	3.054	190.0	113.028	0.56624	0.56697
.019445	3.101	192.9	131.869	.48533	.49345
.022223	3.087	192.1	150.709	.42466	.42980
.027778	3.100	192.9	188.385	.33973	.34529
.033335	3.078	191.5	226.069	.28310	. 28569
.044447	3.074	191.3	301.429	.21233	.21398
.055560	3.082	191.8	376.793	.16986	.17163
.066673	3.101	192.9	452.160	.14155	.14390
.077786	3.084	191.9	527.528	.12133	.12266
. 088900	3.093	192.4	602.901	.10616	.10764
.111130	3.086	192.1	753.658	. 08492	.08594
.133360	3.099	192.9	904.419	.07077	.07191
.177826	3.101	192.9	1,205.975	. 05307	.05396
.222299	3.109	193.4	1,507.578	. 04246	.04327
.266777	3.125	194.4	1,809.221	.03538	.03624
.311265	3.168	197.1	2,110.924	. 03032	.03150

$$L_{T} = 967.099$$
 cm

$$\eta = 62.23104 \frac{\Delta P}{Q_m}$$
, micropoises

$$\frac{64}{Re}$$
 = theoretical friction factor

$$f_{s} = \frac{4r_{m}A^{2}}{\rho L_{T}} \cdot \frac{\Delta P}{Q_{m}^{2}} = 3.094201 \times 10^{-3} \frac{\Delta P}{Q_{m}^{2}}, \text{ experimental friction factor,}$$

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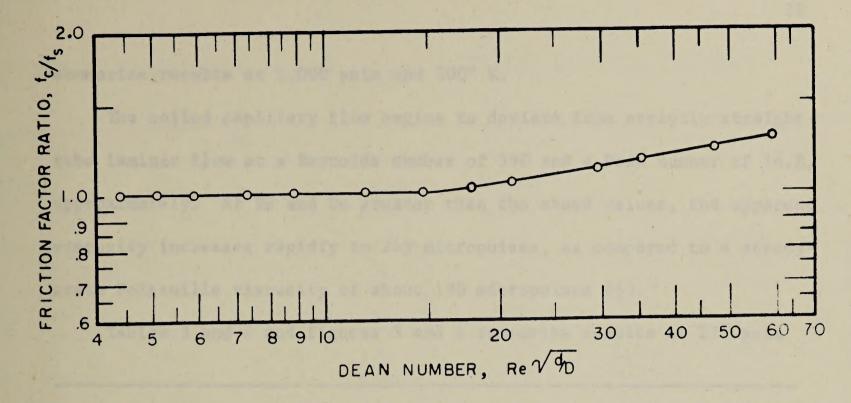


FIGURE 2.—Steady-State Boundary Flow Conditions, 1,000 psia and 300 °K, Friction Factor Ratio Versus Dean Number.

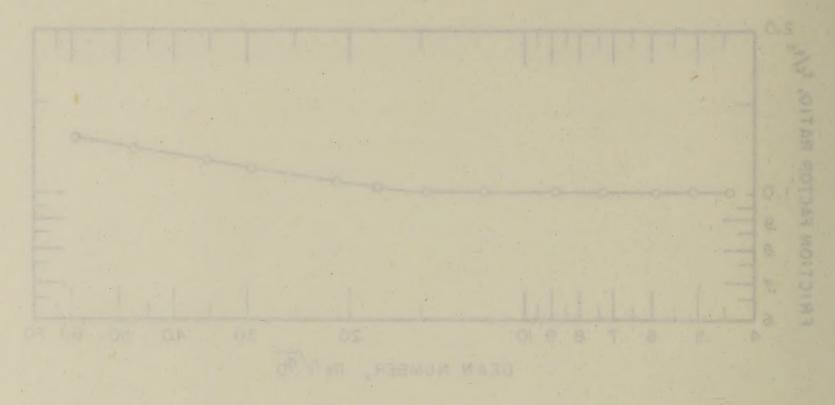


FIGURE 2 - Steady - State Boundary Flow Conditions, 1000 psid and

summarize results at 1,000 psia and 300° K.

The coiled capillary flow begins to deviate from strictly straighttube laminar flow at a Reynolds number of 390 and a Dean number of 14.8,
approximately. At Re and Dn greater than the above values, the apparent
viscosity increases rapidly to 245 micropoises, as compared to a steadystate Poiseuille viscosity of about 190 micropoises (6).

Tables 3 and 4 and figures 3 and 4 summarize results at 225 psia

Figure 3.-Steady-state boundary flow conditions, 225 psia and 300° K, friction factor versus Reynolds number.

Figure 4.-Steady-state boundary flow conditions, 225 psia and 300° K, friction factor ratio versus Dean number.

and 300° K. The friction factor-Reynolds number plot does not show any obvious deviation, mainly because large changes in Re are reflected by small changes in the friction factor. However, the friction factor ratio-Dean number plot deviates at a Dean number of about 14.5 which corresponds to a Reynolds number of about 382. Also, the apparent viscosities in table 3 indicate a change in the type of fluid flow for the regions just mentioned. The coil data deviations at 1,000 and 225 psia should have occurred at the same Reynolds number because of the necessity of maintaining similitude in steady-state laminar flow (8). At the 1,000- and 225-psia pressure levels, it would have been interesting to have obtained higher Reynolds and Dean numbers in order

susmerize results at 1,000 psis and 300° K.

The cotied capillary flow begins to davista from stricely straight. Tube laminar flow at a Maynolds number of 390 and a Dean number of 14.8, approximately. At He and On graster than the above values, the apparent viscosity increases rapidly to 245 micropoises, as compared to a steady-state Poissuille viscosity of about 190 micropoises (6).

Tables 3 and 4 and figures 3 and 4 summerice results at 225 outs

Figure 3.-Stondy-arms boundary flow conditions, 225 pats and 500° K, frietion factor versus Reynolds number.

Figure A. -Steady-state boundary flow conditions, 225 pais and 300° K. friction factor ratio warsus Dean number.

and 300° K. The friction factor-Newholds number plot does not show any obvious deviation, mainly because large changes in Re are reflected by small changes in the friction factor. However, the friction factor ratio-beak number plot deviates at a Dawn number of about 14.5 which corresponds to a Layablds number of about 182. Also, the apparent viscosities in table 3 indicate a change in the type of field flow for the regions just mentioned. The coil data deviations at 1,000 and 225 paid should have occurred at the same Reynolds number because of the necessity of maintaining statificade in steady-state landner flow interesting to have obtained higher Reynolds and Dean numbers da order interesting to have obtained higher Reynolds and Dean numbers da order

TABLE 3.-Experimental results, 225 psia and 300° K, coiled capillary

Q _m , cm ³ /sec	ΔP/Q _m	Apparent viscosity, N, micro- poises	Reynolds number, Re	Dean number, Dn	f _s , 64/Re	f _c	$\frac{f_c/f_s}{}$
0.027782	2.879	179.7	45.015	1.76	1.42175	1.42494	1.0022
.033340	2.879	179.7	54.022	2.11	1.18472	1.18734	1.0022
.044457	2.879	179.6	72.033	2.81	.88848	0.89038	1.0021
.055575	2.879	179.6	90.048	3.52	.71073	.71221	1.0020
.066694	2.879	179.6	108.065	4.22	.59224	.59321	1.0020
.077816	2.879	179.6	126.085	4.93	.50760	.50857	1.0019
.088939	2.878	179.6	144.108	5.63	.44411	.44493	1.0018
.111190	2.878	179.6	180.162	7.05	.35524	.35584	1.0017
.133447	2.886	180.0	216.225	8.46	.29599	.29727	1.0043
.177981	2.894	180.5	288.383	11.28	.22193	.22353	1.0072
.222543	2.912	181.6	360.587	14.11	.17748	.17988	1.0135
.267135	2.950	184.1	432.839	16.94	.14786	.15181	1.0266
.311758	2.993	186.7	505.144	19.77	.12670	.13198	1.0417

$$r_{\rm m} = 0.0152011$$
 inch = 0.0386108 cm

$$L_T = 964.210 \text{ cm}$$

$$\eta = \frac{\pi r_{\text{m}}^4}{8L_{\text{T}}} \cdot \frac{\Delta P}{Q_{\text{m}}} = 62.407802 \frac{\Delta P}{Q_{\text{m}}}, \text{micropoises; } Q_{\text{m}} = Q_1 (1 + \frac{\Delta P}{2P_{\text{m}}})$$

$$\rho = 0.01762 \frac{g}{cm^3}$$
, interpolated from Flynn (6)

Re =
$$\frac{2\rho r_{m}}{\eta^{*} A} Q_{m} = 1620.305 Q_{m}$$
; $A = \pi r_{m}^{2}$, cm^{2} ; $\eta^{*} \approx 179$ micropoises, from Flynn (6)

$$Dn = Re \sqrt{\frac{2r_m}{D}} = 0.039148 \text{ Re}; D = 50.3865 \text{ cm}$$

f = theoretical friction factor, straight tube

$$f_{c} = \frac{4r_{m}A^{2}}{\rho L_{T}} \cdot \frac{\Delta P}{Q_{m}^{2}} = 0.013748 \frac{\Delta P}{Q_{m}^{2}}, \text{ experimental friction factor, coil}$$

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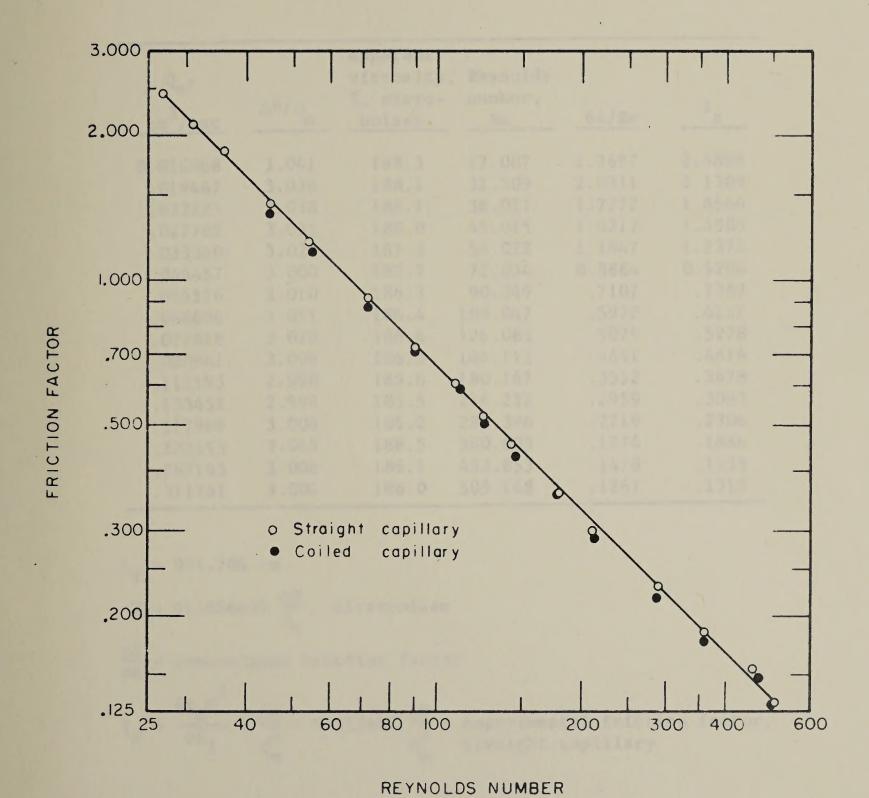
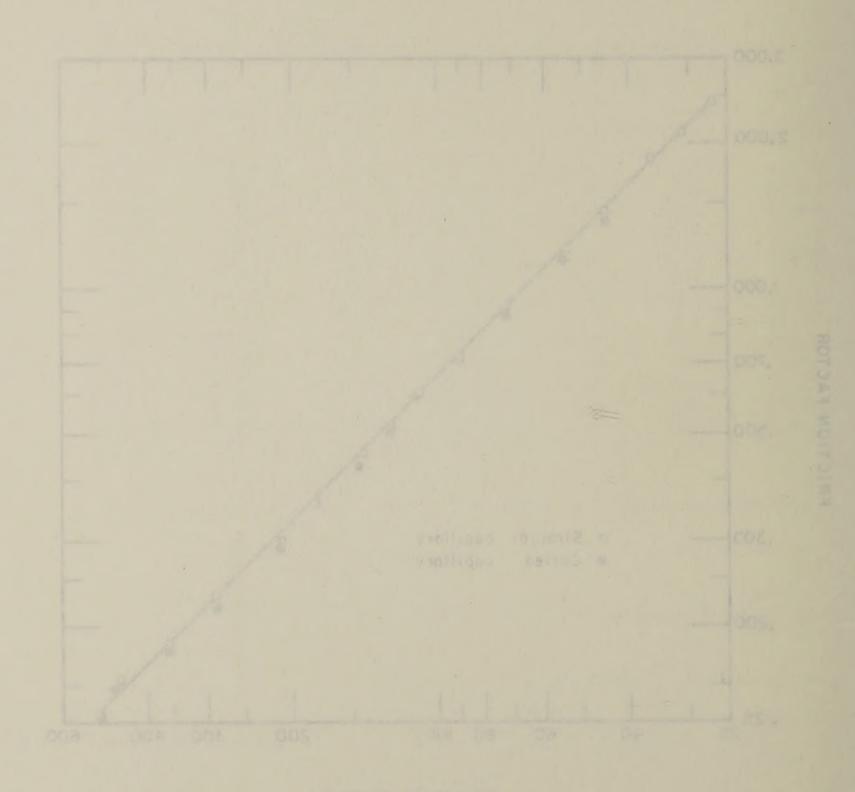


FIGURE 3.—Steady—State Boundary Flow Conditions, 225 psia and 300 °K, Friction Factor Versus Reynolds Number.



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FIGURE 3.-State Boundary From Conditions, 223 psis and 300 %, Fridion Factor Versia Reynolds Number,

TABLE 4.-Experimental results, 225 psia and 300° K, straight capillary

Q _m ,	AR/o	Apparent viscosity N, micro-	, Reynolds		£
cm ³ /sec	$\Delta P/Q_{m}$	poises	Re	64/Re	fs
			07.007	2 2607	2 / 202
0.016668	3.041	188.3	27.007	2.3697	2.4886
.019447	3.038	188.1	31.509	2.0311	2.1309
.022225	3.038	188.1	36.011	1.7772	1.8644
.027782	3.035	188.0	45.015	1.4217	1.4905
.033340	3.024	187.3	54.022	1.1847	1.2374
.044457	3.000	185.7	72.034	0.8884	0.9206
.055576	3.010	186.3	90.049	.7107	.7389
.066696	3.011	186.4	108.067	.5922	.6157
.077818	3.010	186.4	126.085	. 5075	.5278
. 088941	3.009	186.3	144.112	. 4441	.4616
.111193	2.998	185.6	180.167	.3552	. 3678
.133452	2.996	185.5	216.232	. 2959	.3063
.177989	3.006	186.2	288.396	.2219	. 2306
.222553	3.045	188.5	360.603	.1774	.1866
.267143	3.006	186.1	432.853	.1478	.1535
.311761	3.004	186.0	505.148	.1267	.1315

$$L_{T} = 971.704 \text{ cm}$$

$$\eta = 61.926499 \frac{\Delta P}{Q_m}$$
, micropoises

$$\frac{64}{Re}$$
 = theoretical friction factor

Re
$$f_{s} = \frac{4r_{m}^{A^{2}}}{\rho L_{T}} \cdot \frac{\Delta P}{Q_{m}^{2}} = 0.013642 \frac{\Delta P}{Q_{m}^{2}} = \text{experimental friction factor,}$$

$$g_{m}^{A} = \frac{4r_{m}^{A^{2}}}{\rho L_{T}} \cdot \frac{\Delta P}{Q_{m}^{2}} = 0.013642 \frac{\Delta P}{Q_{m}^{2}} = \text{experimental friction factor,}$$

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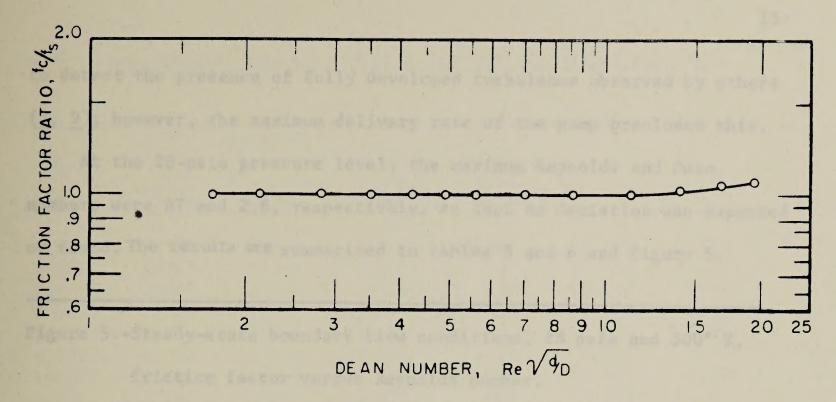
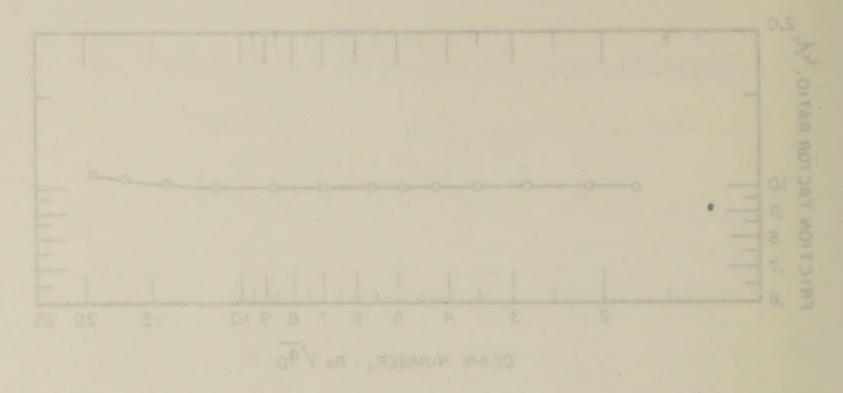


FIGURE 4.—Steady—State Boundary Flow Conditions, 225 psia and 300 °K, Friction Factor Ratio Versus Dean Number.



FrauRE 4.-Steam, State Boundary Flow Conditions, 225 psig and 300 °K, Friction Factor Ratio Versus Dean Number.

to detect the presence of fully developed turbulence observed by others (3, 9); however, the maximum delivery rate of the pump precluded this.

At the 28-psia pressure level, the maximum Reynolds and Dean numbers were 67 and 2.6, respectively, so that no deviation was expected or found. The results are summarized in tables 5 and 6 and figure 5.

Figure 5.-Steady-state boundary flow conditions, 28 psia and 300° K, friction factor versus Reynolds number.

At the start of some particular runs, unexplained variations in $\Delta P/Q_m$ were observed as well as variations of about two to three percent in the computed apparent viscosity, $\eta.$ At the beginning of these, the slowest volumetric flowrates were being used and, apparently, the fluid flow system had not reached steady-state conditions. Also, the temperature control was not as precise as was desired; this was especially true during the straight-tube-configuration runs where the tubing was located at the laboratory ceiling.

CONCLUDING STATEMENTS

It is evident that at a Reynolds number greater than about 390 or at a Dean number larger than about 15, strictly steady-state Poiseuille laminar flow was measurably affected by Dean's circulation at 1,000 and 225 psia. The same behavior would have been observed at 28 psia if higher volumetric flowrates had been available and used. In any case, the data indicate that if one chooses a conservative upper limit of 10

to detect the presence of fully developed turbulence observed by others
(2, 2); however, the maximum delivery rate of the pump precluded this.

At the 28-paid pressure level, the maximum Raynolds and Dean numbers were 67 and 2.0, respectively, so that no declation was expected or found. The results are summarized in cables 2 and 6 and figure 3.

Figure 5.-Steedy-stars boundary flow conditions, 25 pais and 300° E. friction factor versus devnotes number.

At the start of some particular runs, unexplained variations in AP/Om were observed as well as variations of about two to three perrent in the computed apparent viscosity, W. At the beginning of these, the slowest volumetric flowrotes were bring used and, apparently, the fluid flow system had not reached standy-state conditions. Also, the temperature control was not as greeise as was domined; this was especially true during the straight-tube-configuration runs where the toolog was located at the laboratory calling.

COVICE UD FING STATEMENTS

It is evident that at a Seynolds number greater than about 900 or at a Dean number larger than about 15, strictly steady-atate Poissuille laminar flow was measurably affected by Dean's pironistion at 1,000 and 125 pale. The same behavior would have been observed at 28 pale it higher volumetric flowrates had been available and used. In any case, that date indicate that if one chooses a domestive upper limit of 10

TABLE 5. - Experimental results, 28 psia and 300° K, coiled capillary

		Apparent				,	
Q _m ,		viscosity,	Reynolds	Dean			
	ΔP/Q _m	N, micro-	number,	number,	f 6/1/Ra	£	f /f
cm ³ /sec	m	poises	Re	Dn	f _s , 64/Re	f _c	c's
0.010/62	0 775	372 1	/ 205	0.167	15 010	1/ 001	0.074
0.019462	2.775	173.1	4.205	0.164	15.219	14.821	0.974
.022246	2.787	173.9	4.807	.187	13.313	13.024	.978
.027815	2.804	175.0	6.010	. 233	10.648	10.481	.984
.033388	2.785	173.8	7.214	.280	8.872	8.673	.978
:044542	2.784	173.7	9.624	.374	6.650	6.497	.977
.055709	2.782	173.6	12.037	.468	5.317	5.192	.976
.066889	2.796	174.5	14.444	.562	4.431	4.345	.981
.078080	2.792	174.2	16.872	.657	3.793	3.717	.980
.089284	2.789	174.0	19.292	.750	3.317	3.247	.979
.111731	2.783	173.7	24.129	.939	2.652	2.590	. 977
.134229	2.786	173.9	28.988	1.128	2.208	2.158	.977
.179378	2.787	173.9	38.762	1.508	1.651	1.615	.978
. 2247 34	2.786	173.8	48.537	1.888	1.318	1.288	. 97 7
.270315	2.797	174.5	58.418	2.272	1.096	1.076	.982
.316055	2.772	173.0	68.270	2.655	0.937	0.912	.973

$$r_{\rm m} = 0.0152009$$
 inch = 0.0386103 cm

$$L_{\rm T} = 964.210~{\rm cm}$$

$$\eta = \frac{\pi r_{\text{m}}^{4}}{8L_{\text{T}}} \cdot \frac{\Delta P}{Q_{\text{m}}} = 62.404571 \frac{\Delta P}{Q_{\text{m}}}, \text{ micropoises; } Q_{\text{m}} = Q_{1}(1 + \frac{\Delta P}{2P_{\text{m}}})$$

$$\rho = 0.00233 \frac{g}{cm}$$
, extrapolated from Flynn (6)

Re =
$$\frac{2\rho r_m}{\eta^* A} Q_m = 216.073 Q_m$$
; A = πr_m^2 , cm²; $\eta^* \approx 177$ micropoises, from Flynn (6)

$$Dn = Re \sqrt{\frac{2r}{D}} = 0.038890 Re; D = 50.3865 cm$$

f_s = theoretical friction factor, straight tube

$$f_{c} = \frac{4r_{m}^{A^{2}}}{\rho L_{T}} \cdot \frac{\Delta P}{Q_{m}^{2}} = 0.103959 \frac{\Delta P}{Q_{m}^{2}}, \text{ experimental friction factor, coil}$$

		,		
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Ly = 966,210 cm

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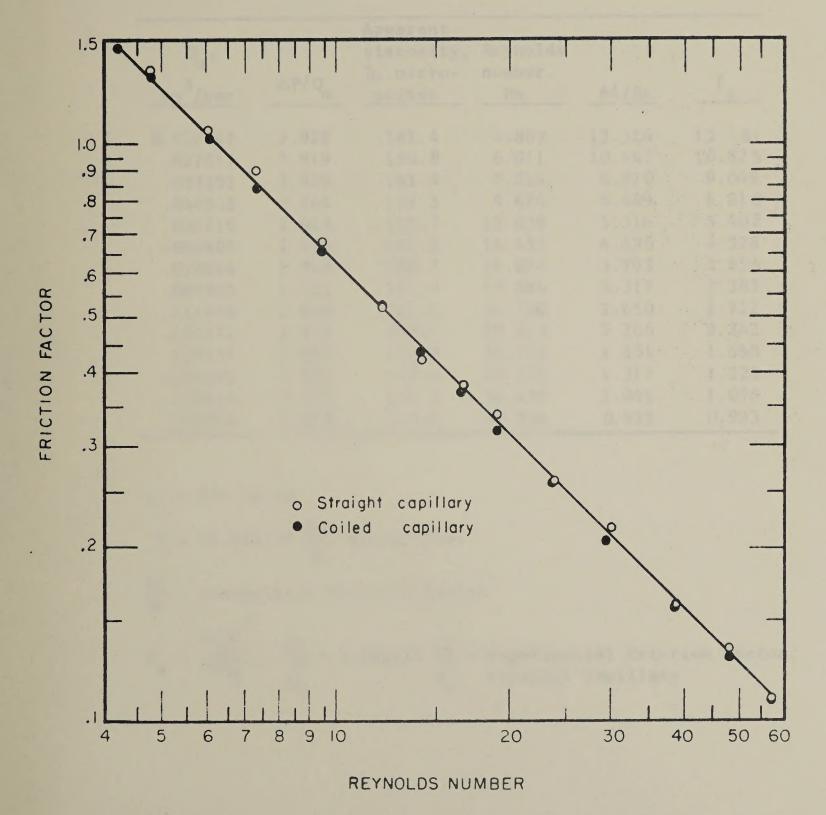
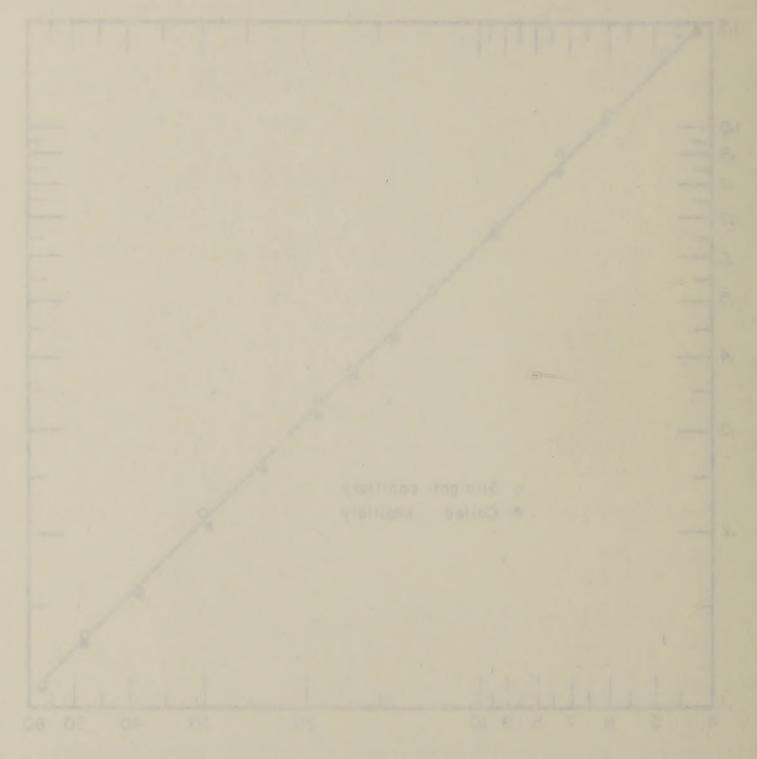


FIGURE 5.—Steady—State Boundary Flow Conditions, 28 psia and 300 °K, Friction Factor Versus Reynolds Number.



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FIGURE 5.-State Houseony Flow Conditions, 28 psid and 300°K, Grigtion Factor Versus Reynolds Number.

TABLE 6.-Experimental results, 28 psia and 300° K, straight capillary

		Apparent	Dannal da		
Q_{m}		viscosity,	Reynolds		
3	ΔP/Q	η, micro-	number,		f
cm ³ /sec	m	poises	Re	64/Re	S
0.022247	2.929	181.4	4.807	13.314	13.581
.027817	2.919	180.8	6.011	10.647	10.825
.033391	2.929	181.4	7.215	8.870	9.048
.044548	2.944	182.3	9.626	6.649	6.818
.055716	2.918	180.7	12.039	5.316	5.402
.066900	2.936	181.8	14.455	4.428	4.528
.078094	2.918	180.7	16.874	3.793	3.855
.089305	2.929	181.4	19.296	3.317	3.383
.111766	2.938	181.9	24.150	2.650	2.712
.134272	2.918	180.7	29.013	2.206	2.242
.179437	2.888	178.8	38.771	1.651	1.660
.224825	2.884	178.6	48.579	1.317	1.323
.270414	2.870	177.7	58.429	1.095	1.095
.316216	2.859	177.0	68.326	0.933	0.933

$$L_T = 971.04$$
 cm

$$\eta = 61.923293 \frac{\Delta P}{Q_m}$$
, micropoises

$$\frac{64}{Re}$$
 = theoretical friction factor

$$f_{s} = \frac{4r_{m}A^{2}}{\rho L_{T}} \cdot \frac{\Delta P}{Q_{m}^{2}} = 0.103158 \frac{\Delta P}{Q_{m}^{2}} = \text{experimental friction factor,}$$

$$straight capillary$$

	,	

The stranger of the contract of secretary friends factor.

for the Dean number, Dean's circulation will be undetectable, within the experimental accuracies reported here.

In the works of White $(\underline{13})$, Creutz $(\underline{3})$, and Ito $(\underline{9})$, the friction factor ratio versus Dean number plots all strongly suggest an upper limiting Dean number of about 10 for strictly steady-state Poiseuille flow.

This suggests that as long as

$$f_c/f_s = 1 \tag{2}$$

and

Re
$$(d/D)^{1/2} \le 10$$
, (3)

one obtains steady-state laminar flow. It seems reasonable that for coiled systems in general, equation (3) holds. This presumes that the capillary tubing cross sections are not appreciably deformed by winding. Creutz (3) found a 94.5-percent reduction in the mean hydraulic radius in his tighter helices where D/d was in the order from 10 to 15. Our D/d was 660.

for the Dean number, Dean's circulation will be underscribble, whinly

In the world of White (13), Ground (1), and Ito (2), the Eriction factor ratio varsus Dean number plots all strongly anguest as upper limiting hear number of smout IO for strictly strady-state Poissoille

This suggests that as long as

(2)

bns

 $Re^{-(a/0)^{1/2}} \le 10^{--(a/0)}$

one obtains stoudy-state landar flow. It seems resemble that for colled averens in general, savation (3) holds. This presumes that the captillary tobing cross sacilots are not oppositively deformed by winding. Creuts (3) found a whist-persent reduction in the mean hydratile reduction in the cighter lead to 15. Our in the was fed.

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